Profile For Systematic Testing

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You are probably familiar with this: You have to solve a math text problem and just don`t know how to go about it. So called "**problem solving strategies**" might be offering some guidance to crack problems. **Systematic testing** is one of those strategies introduced and made use of in this profile.

Problem 1: Change

How many possibilities are there to change 1 Euro into 5 and 10 cent pieces, with every coin being used at least once?

figure 1: systematic testing – change

Look at the solution only after trying to solve the problem yourself.

With this problem it would seem best to test all possibilities to change 1 Euro into 5 and 10 cent pieces.

possibility	amount of 10 cent pieces	amount of 5 cent pieces
1	1	18
2	2	16
3	3	14
4	4	12
5	5	10
6	6	8
7	7	6
8	8	4
9	9	2

At first we start with a 10 cent piece. This leaves us with a difference of 90 Cent from one Euro. This is made up by 18 5 cent pieces. Next we increase the number of 10 cent pieces by one each time and determine the missing amount of 5 cent pieces. This shows that there are 9 possibilities to change 1 Euro into 10 and 5 cent pieces.

Our strategy on this problem was to vary one of the searched for values systematically and to adjust the second one accordingly. This systematic approach simplifies problem solving and gives an overview of the already used possibilities. It makes sense to tackle similar problems by systematical testing. It requires some consideration beforehand, how the information given can be structured to your advantage.

> **Please note:**
> When dealing with systematic testing you deal with **case differentiation**.

Problem 2: Key

Y friend wants you to get his bathing things from his home. As she hands you the keys she forgets to mention which key unlocks the front door. Because there are about 15 keys on the bunch of keys and you really want to hurry to get to the lake, you wonder, how you will get the bathing things as quick as possible. Look for features that will help you find the right key and neglect the wrong ones.

figure 2: systematic testing – key

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3rd*1st size of the key*
2nd...
3rd...

You only have to put those keys in the lock that might be relevant keys. That way you have reduced the effort and are now able to reach your goal faster.

> **Please note:**
> - Put information into an advantageous order
> - draw up a shortlist of possible solutions
> - * try effectively

These examples show what systematic testing is all about and where it can sometimes be helpful to go about it systematically. Often systematic testing does not lead to an end in problem situations. Additional heuristic means can be of help - like a chart, an informative figure and equation - to reduce the situation and to visualize the problem. The following example shows how information can be put in order and recorded by a chart. In the same way as with the systematic testing, you can see where a possible solution could lead.

Problem 3: Tiles
Family Miller wants to tile a rectangular kitchen with even square tiles At the hardware store they find a sale of remainders with 32 dark tiles and 48 bright tiles. "That fits well, if we do the bright tiles in the middle and the darker on the outside." says the father. How many tiles have to be laid in width and length?

length	width	result	
inner tiles		outer tiles	works
1	48	102	no
2			

> **Advantages of a chart:**
> - clarity
> - helps to not forget a possibility
> - quick recognition of systematic in the results

You may apply the newly learned problem solving strategies with the following problem. Document your solution process!

Problem 4: Candles
Two candles burn down at varying speeds: Candle A has a length of 36cm and burns with 3cm an hour, candle B is has a length of 10cm and burns 1cm per hour. When will both candles have the same length?

Which solution process did you choose? Did you test systematically?

Alternatively you could also come up with an equation or an informative figure. Solve the problem by means of an **equation** or an **informative figure**, if you did not chose that solution process. Look at the presented procedure of solution. Do you notice anything?
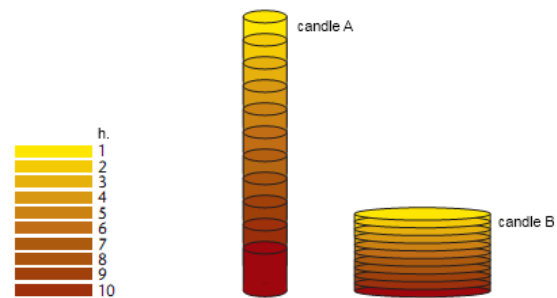
Solution process with the help of an informative

figure 3: systematic testing – candles

> **Advantages of the means informative figure:**
> - "drawing" for a better understanding
> - contains all important information
> - visualizing solutions

Procedure by means of equation:

candle A: y = 36 - 3x
candle B: y = 10 –x

By inserting reasonable values in both equations you come to the conclusion, that both candles reach the same length after 13 hours.

At this point of processing the problem you should check the reality of the situation, because after 13 hours burning time the candles have reached a length of -3cm! This problem shows how hasty work with equations can make you hit a wall at times, if no additional graphic context or chart is lined up to be paying attention to the real situation (positive length of candles) of a mathematical model. A graphic context or systematic trying by means of a chart shows, that both candles have reached their common length, which is 0 after 12 hours!

> **Advantages of the means equation:**
> - link between information
> - problem can be simplified

Even in daily life systematic testing can be helpful. For example if you want to boil some eggs for breakfast and you don`t know how long an egg has to boil until it has the right consistency and you are on your own, there is nothing else for you to do but o check it out. Not just somehow, according to the trial-and-error technique, but systematically. If you come up with some kind of system, you will quickly discover how long an egg has to boil. You will maintain clarity, if you work with charts too.
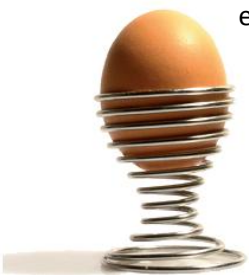
figure 4: systematical testing - egg

You solved some problems by systematical testing already. Which situations (math class, other subjects, daily life) did you already apply the strategy of systematical testing? Where can you do so?

Exercises

The following problems give you the opportunity to apply the learned strategies about handling problems, especially the systematical testing and to gain experience in solving problems.
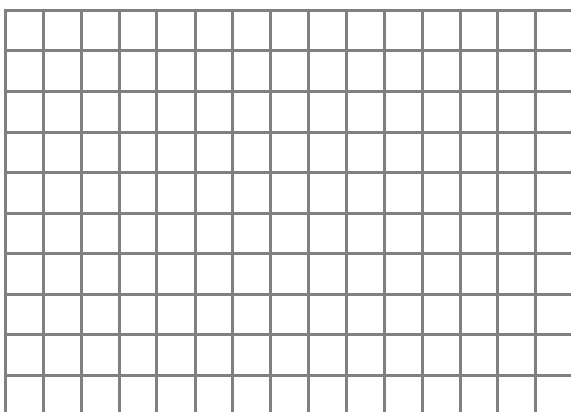
Problem 5: Paper

A piece of paper is given. It will be cut in either 8 or 12 pieces of any size. Every single one of the resulting pieces can be cut again into either 8 or 12 pieces etc.

a) Is it possible to get 60 pieces that way?
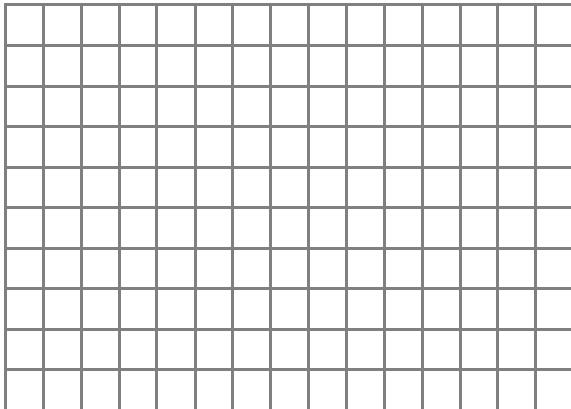
b) Look at your solution process. What do you notice about optional numbers of pieces greater than 60?

Problem 6: Hats

Experiments were carried out to make math class more interesting. One time the teacher showed 5 hats. Three were black and two were red. Three students (Horst, Bernd and Ulrike) were seated on chairs in a row. The room was completely darkened and the teacher put three hats on their heads. The light was switched back on. None of the three was allowed to turn around. Horst only saw the hats of Bernd and Ulrike, Bernd only saw Ulrike's hat and Ulrike saw no other. Now Horst was asked, if he was wearing a red or a black hat. He said, "I cannot say for sure." Bernd, hearing Horst`s answer, answered the same as Horst. Drawing her conclusions Ulrike was able to tell the color of her hat.

Which color was it?

figure 5: systematical testing: hats
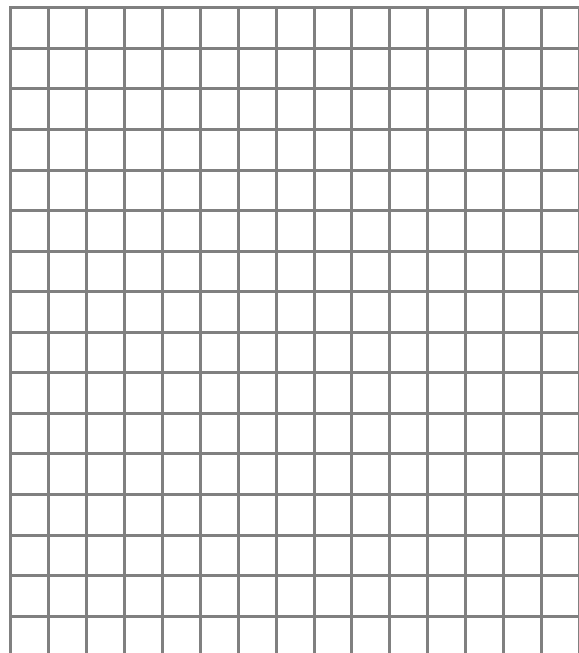
Problem 7: Safe

You are given the chance to participate at a raffle at a mall. To get to your prize you need to enter a 10-digit code you know the following about:

- *The one-digit code beginning from the left can be divided by 1.*
- *The two-digit code beginning from the left can be divided by 2.*
- *The three-digit code beginning from the left can be divided by 3.*
- *The four-digit code beginning from the left can be divided by 4.*
- *The five-digit code beginning from the left can be divided by 5.*
- *The six-digit code beginning from the left can be divided by 6.*
- *The seven-digit code beginning from the left can be divided by 7.*
- *The eight-digit code beginning from the left can be divided by 8.*
- *The nine-digit code beginning from the left can be divided by 9.*
- *The entire code beginning from the left can be divided by 10.*

What is the code that opens the safe if every number can only be used once?
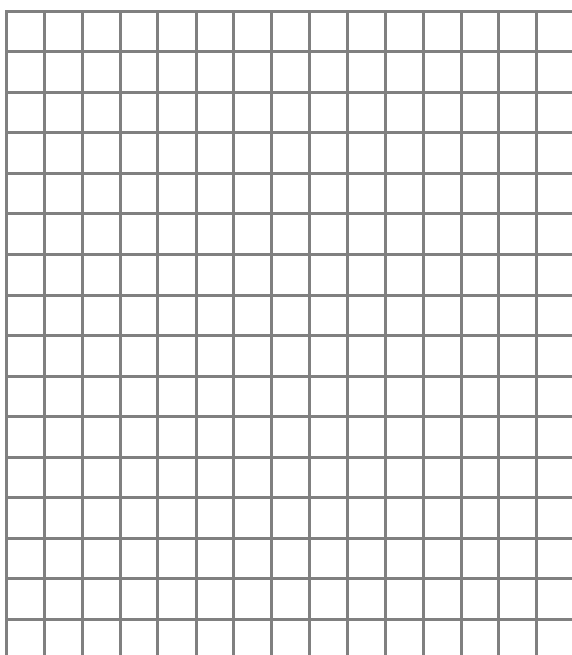
Problem 8: Age of the bishop

A pastor told the organist: "Today there were only three people at church."
Organist: "How old were those three?"
Pastor: "If their ages are multiplied it is 2450. Added up they are as old as you."
Organist: Hmm, with this information I cannot solve the problem!"
Pastor: "Oh well, I have to also mention that the three of them are younger than our bishop!"
Organist: "Aha, now I have got it!"
How old is the bishop?
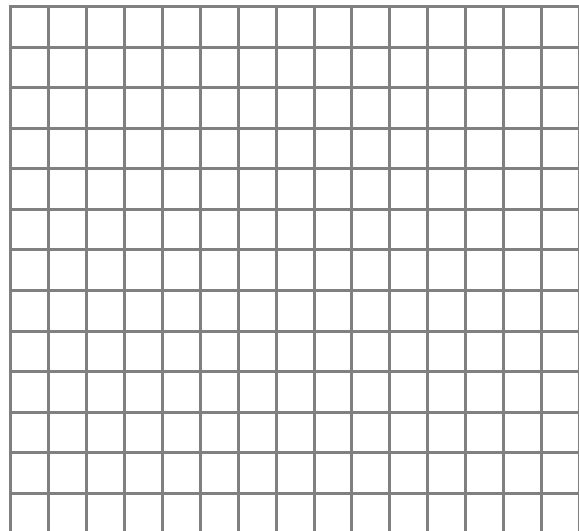
Conclusion

Testing is often one of the first activities in search ing for a solution. This method is not "unmathematical". Many mathematicians use this strategy to get an idea of the solution they want to prove. Systematic testing by various means (chart, informative figure, equation) can be summarized: First all relevant information is gathered, ordered and - if possible - tested systematically for satisfying cominations. This strategy does not necessarily proide an exact solution, but can be very helpful along the way of finding a solution.

Solutions

Problem 3: Tiles

length	width	result	
inner tiles		outer tiles	works
1	48	102	✗
2	24	56	✗
3	16	42	✗
4	12	36	✗
6	8	32	✓
8	6	32	✓
12	4	36	✗
16	3	42	✗
24	2	56	✗
48	1	102	✗

Problem 4: Candles - alternative solution

candle A: y = 36 - 3x, candle B: y = 10 - x

The two equations are equated with each other the solution is reached, that both candles will have the same length after 13 hours. After 12 hours they already reach the same length, which is 0cm.
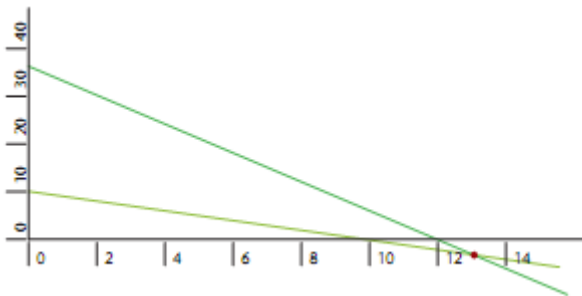
Problem 5: Paper

	0	1	2	3	4	5	6	7	8	9	10
0	1	8	15	22	29	36	43	50	57	**64**	71
1	12	19	26	33	40	47	54	**61**	68		
2	23	30	37	44	51	58	**65**	72			
3	34	41	48	55	**62**	69					
4	45	52	59	**66**	73						
5	56	**63**	70								
6	**67**	74									

This chart presents all possible amounts of pieces up to 74. The number 60 never occurs. Because all numbers from 61 up to 67 are given in the chart it is enough to move on to the right to get any amount of pieces bigger than 60.

Problem 6: Hats

all possible combinations

Horst	Bernd	Ulrike	Horst is unsure	Bernd is unsure
🔴	🔴	⚫		
🔴	⚫	🔴		no
🔴	⚫	⚫		
⚫	🔴	🔴	no	
⚫	🔴	⚫		
⚫	⚫	🔴		no
⚫	⚫	⚫		

The first answer tells us that Bernd and Ulrike cannot have both two red hats on (so Horst would have had a black one). So Ulrike must be wearing a black hat.

Problem 7: Safe

We know the code is made out of ten digits:

X X X X X X X X X X

The entire code has to be divisible by 10 so only a 0 is the possibility for the last digit. The fifth digit from left can be divisible by 5, which is the case for numbers ending in a zero or a five. Because the zero is already taken only a 5 remains:

X X X X 5 X X X X 0

The 2nd, 4th, 6th and 8th digit need to be even, because they can be divided by 2,4,6 and 8. thus the 1st, 3rd, 7th and 9th digit are uneven.

By systematical testing and eliminating you get the following combinations:

1	2	3	6	5	4	n/a			
1	2	9	6	5	4	7	n/a		
1	4	n/a							
1	6	7	2	5	n/a				
1	8	3	2	5	n/a				
1	8	3	6	5	4	n/a			
1	8	9	2	5	n/a				
1	8	9	6	5	4	n/a			
3	2	1	6	5	4	9	n/a		
3	2	7	6	5	4	n/a			
3	4	n/a							
3	6	9	2	5	8	n/a			
3	8	1	2	5	n/a				
3	8	1	6	5	4	7	2	9	0
3	8	7	2	5	n/a				
3	8	7	6	5	4	n/a			
7	2	n/a							
7	4	1	2	5	8	n/a			
7	4	1	6	5	n/a				
7	6	n/a							
7	8	3	2	5	n/a				
7	8	9	6	5	4	n/a			
9	2	1	6	5	4	3	n/a		
9	2	7	6	5	4	n/a			
9	4	n/a							
9	6	3	2	5	8	1	n/a		
9	8	1	2	5	n/a				
9	8	1	6	5	4	n/a			
9	8	7	2	5	n/a				
9	8	7	6	5	4	n/a			

So the solution is:
3 8 1 6 5 4 7 2 9 0

Problem 8: Age of the bishop

The age of the three church attendants shall be presumed as natural and integer. By testing you obtain 20 different cases:

	1st age	2nd age	3rd age	sum
1.	1	1	2450	2452
2.	1	2	1225	1228
3.	1	5	490	496
4.	1	7	350	358
5.	1	10	245	256
6.	1	14	175	190
7.	1	25	98	124
8.	1	35	70	106
9.	1	49	50	100
10.	2	5	245	252
11.	2	7	175	184
12.	2	25	49	76
13.	2	35	35	72
14.	5	5	98	108
15.	5	7	70	82
16.	5	10	49	64
17.	5	14	35	54
18.	7	7	50	64
19.	7	10	35	52
20.	7	14	25	46

Because the organist was not able to clearly solve the problem with the given information, he is 64 years old. Only when the sum is 64, the product isn`t clear.

The additional information states, that the bishop must be older than 49 years. If the bishop would be 51 or older both possibilities for the age of the church attendants would apply and it would again be unclear. But because the problem should be solvable now, the bishop can only have the age of 50 years. Then only the one solution for the church attendants to be 5 years, 10 years and 49 years old applies. Only in this case all attendants are younger than the bishop. The bishop is 50 years old.

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Meyer W. (2002): Lösungsstrategien für mathematische Aufgaben. Köln: Aulis Verlag Deubner.

Register of illustrations

figure 1: systematical testing – change
photographer: Kellermeister (www.pixelio.de)

figure 2: systematical testing – key
photographer: U. Herbert (www.pixelio.de)

figure 4: systematical testing – egg
photographer: fult (www.photocase.de)

figure 5: systematical testing – hat red: www.xxxdrive.eu black: www.logotextilien.at

figure 6: systematical testing – safe www.werkstatt-werksverkauf.de

Directory of problems

Problem 1
material Aufgabenpraktikum online, TUD, WS 07/08

Problem 3
available at: http://www.standardsiche-rung.nrw.de/material-datenbank/upload/ 301/f1119973076046_E_6_15_ Systematisches%20 Probieren.doc (last checked: 09.05.2008)

Problem 4
Büchter, A. & Leuders, T. (2005): Mathematikaufgaben selbst entwickeln. Lernen fördern – Leistung überprüfen. Berlin: Cornelsen Scriptor, page 18

Problem 5
Meyer W. (2002): Lösungsstrategien für mathematische Aufgaben. S. 106f. Aulis Verlag Deubner & Co. KG, Köln, S. 106f.

Problem 6
material Aufgabenpraktikum online, TUD, WS 07/08

Problem 7
material Fachdidaktisches Projekt Problemlösen, TUD, SS 2008

Problem 8
available at: http://www.brefeld.home-page.t-online.de/bischof.Html (last checked: 18.08.2008)